Analysis of Memristor Behavior in presence of Harmonics

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***Abstract*—Memristor is the fourth basic passive circuit element after resistor, capacitor and inductor. It was proposed in 1971 by Leon Chua using symmetry arguments but was clearly experimentally demonstrated in 2008. Since then a number of models for the device have been proposed, this paper reviews two of these models namely Numerical Model and Fully Symbolic Homotopy based Model. Further, these two models are used to analyze the behavior of Memristor in presence of voltage and current time harmonics in addition to the fundamental component.**

***Index Terms*—Memristor, Mathematical Modelling** *(key words)*

INTRODUCTION

Memristor is the fourth basic passive circuit element, it was proposed by Leon Chua in 1971[1]. Each one the existing passive circuit elements i.e. R,L and C link two out of the four basic circuit variables i.e. V, I, Q and Φ. Where
V - Voltage, I – Current, Q- Charge and Φ - Flux.
So, a hypothetical device was formulated relating the only remaining pair of charge and flux. Researchers at HP labs realized an actual physical memristor in 2008[2].

Memristor realized by HP is represented by two coupled variable resistors having resistances

$R_{ON}\,{}^{w}/_{D}$ And $R_{OFF}\left(1-{}^{w}/_{D}\right)$

Where w is function of current

$$w(t) = \mu_V\,{}^{R_{ON}}/_{D}\int_0^t i(\tau)\,d\tau \qquad (1)$$

So it's total resistance or as it called memristance is given by

$$M(t) = R_{ON}\,{}^{w(t)}/_{D} + R_{OFF}\left(1-{}^{w(t)}/_{D}\right) \qquad (2)$$

Solving these two equations, a mathematical model of memristor is developed. Numerical model is obtained by solving simply the two equation. Solution by Homotopy Method leads to a fully symbolic model.

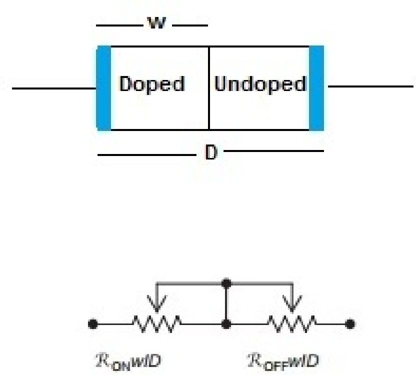

Fig.1 Coupled variable Resistor Model

NUMERICAL MODEL

Width w(t) of the doped material is determined by the quantity of current flown. It can be expressed as the function of charge as

$$w(t) = \mu_V\,{}^{R_{ON}}/_{D}\,q(t) + w_0 \qquad (3)$$

Where w_0 is the width of the doped material at time zero.

Solving eqns. (2) and (3) Memristance can be written as a function charge q as

$$M(q) = M_0 - \left({}^{\mu R_{ON} R_D}/_{D^2}\right) q(t) \qquad (4)$$

Where

$$M_0 = R_{ON}\,{}^{w_0}/_{D} + R_{OFF}\left(1-{}^{w_0}/_{D}\right) \qquad (5)$$

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Memristance as a function of flux is

$$M^2 = {M_0}^2 - \left(\frac{2\mu R_{ON} R_D}{D^2}\right)\varphi(t) \qquad (6)$$

M is bounded by R_{ON} and R_{OFF}, i.e.

$M \in (R_{ON}, R_{OFF})$

And

$$\varphi(t) = \int_0^t v(\tau)d(\tau) \qquad (7)$$

Using eqns. (6) and (7), Memristor response for any voltage signal can be analyzed. So, this provides as a complete mathematical model for Memristor.

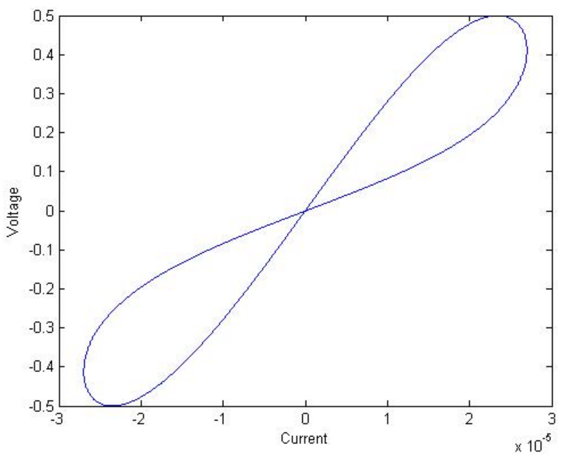

Fig.2 IV characteristics of Memristor obtained by simulating the Numerical Model in MATLAB

FULLY SYMBOLIC MODEL

A model for memristor was obtained by solving the differential equation, governing the physical functioning of the device, by using a homotopy formulation in [3].

Homotopy Perturbation Method was used to obtain a model of memristor when only a sinusoidal current signal is applied and it resulted in a form [3]

$$M = (\alpha - 1)\left(\begin{array}{c} \left[\begin{array}{c} -\frac{3}{4}\left(\frac{\mu {A_p}^2 a_1}{D^2\omega}\right)^2 X_0\left(5{X_0}^2 - 1\right)(X_0 - 1)^3(X_0 + 1)^3 R_{ON}^2 \\ -\left(\frac{\mu A_p a_1}{D^2\omega}\right) X_0 (X_0 - 1)^2 (X_0 + 1)^2 R_{ON}^2 \\ + X_0 \frac{[1 + (X_0^{-1} - 1)\alpha]}{(\alpha - 1)} R_{ON} \end{array}\right] \\ + \left[\begin{array}{c} \left(\frac{\mu {A_p}^2 a_1}{D^2\omega}\right)^2 X_0 (5X_0^2 - 1)(X_0 - 1)^3 (X_0 + 1)^3 R_{ON}^2 \\ + \left(\frac{\mu A_p a_1}{D^2\omega}\right) X_0 (X_0 - 1)^2 (X_0 + 1)^2 R_{ON}^2 \end{array}\right]\cos(\omega t) \\ + \left[\frac{1}{4}\left(\frac{\mu {A_p}^2 a_1}{D^2\omega}\right)^2 X_0\left(5{X_0}^2 - 1\right)(X_0 - 1)^3 (X_0 + 1)^3 R_{ON}^2\right]\cos(2\omega t) \end{array}\right)$$

With this as basis in the next section a model is developed where current signal can take any general form and a mathematical expression is provided for w(t) derived using HPM method for a case where an Additional component is present along with the fundamental one.

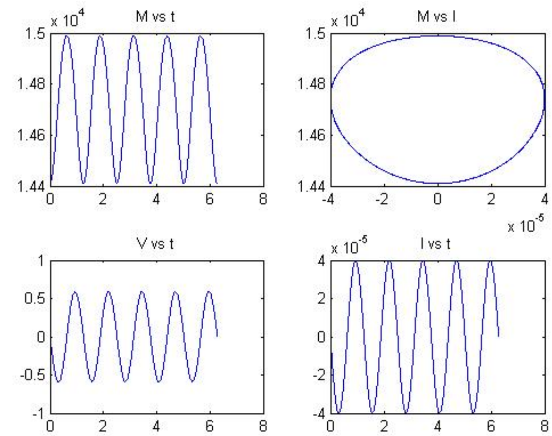

Fig.3 The various plots obtained by simulating the Fully Symbolic Homotopy based Model in MATLAB.

BEHAVIOR IN PRESENCE OF HARMONICS

In this section the response of memristor is studied in presence of time harmonics, additional to the fundamental voltage and current components. Simulation experiments were independently carried on the two models using MATLAB.

For Current Harmonics Fully Symbolic Homotopy Based Model was used.

In general terms w(t) obtained from Fully Symbolic Model can be written as

$$w(t) = A + B\int_0^t i(\tau)d\tau + C\int_0^t i(\tau)\theta(\tau)d\tau \qquad \ldots(8)$$

Where

$$A = X_0$$
$$B = -\delta X_0 (X_0 - 1)^2 (X_0 + 1)^2$$
$$C = \delta^2 X_0 (5{X_0}^2 - 1)(X_0 - 1)^3 (X_0 + 1)^3$$

And

$$\theta(t) = \int_0^t i(\tau)d\tau \qquad (9)$$

So for two components let

$$i(t) = A_{p1} sin\omega t + A_{p2} sin 2\omega t$$

$$\theta(t) = \frac{1}{\omega}\left(A_{p1}cos\omega t + \frac{A_{p2}}{2}cos2\omega t - (A_{p1} + \frac{A_{p2}}{2})\right)$$

$$\int_0^t i(\tau)\theta(\tau)d\tau = \frac{1}{\omega^2}\left(\frac{{A_{p2}}^2}{8}(cos4\omega t - 1) + \frac{A_{p1}A_{p2}}{4}(cos3\omega t - 1) + \frac{1}{2}\left(\frac{{A_{p1}}^2}{2} - \frac{{A_{p2}}^2}{2} - A_{p1}A_{p2}\right)(cos2\omega t - 1) - \left({A_{p1}}^2 + \frac{A_{p1}A_{p2}}{4}\right)(cos\omega t - 1)\right) \quad (10)$$

The highest frequency term is four times the fundamental component. If there are n components then the highest frequency term will be 2n times the fundamental.

For more than two components analysis using Homotopy based method becomes very cumbersome and time consuming. So, simulations were carried out for current signal having only two components. R_{ON}=100Ω, a=160,X_0=0.1,D=10^{-8}m, μ=$10^{-10}cm^2s^{-1}V^{-1}$

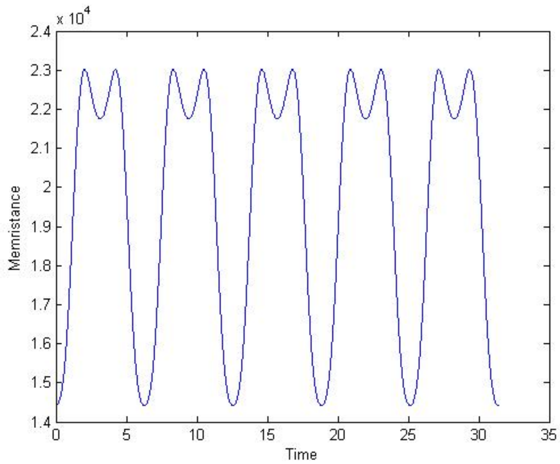

Fig.4 Memristance as a function of time when one additional harmonic is present

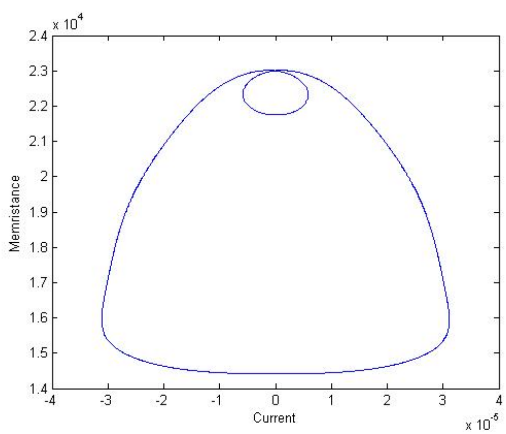

Fig.5 Current Vs Memristance

Using Numerical Model the memristor behavior was studied when only Voltage harmonics are present. This expression is presented for the first time in this paper.

$${M_{eff}}^2 = M^2 + {M_1}^2 - {X_0}^2 \quad (11)$$

Where

M is the resistance at time t when only Fundamental component is applied.

M1 is the resistance at time t when only first harmonic is applied.

M_{eff} is the resistance at time t when both fundamental and first harmonic are applied simultaneously.

For in general

$$M(t)_{eff}^{\ 2} = M(t)^2 + M_1(t)^2 + M_2(t)^2 \ldots\ldots\ldots.. M_n(t)^2 - n{X_0}^2$$
(12)

Where M_n is the resistance when only nth harmonic is applied.

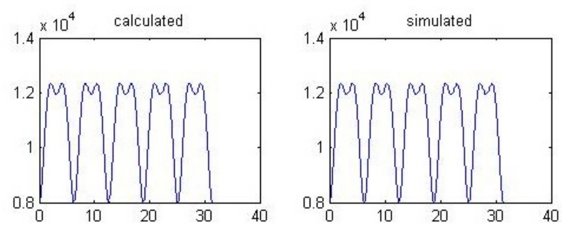

Fig.6 Calculated and simulated values of Memristance vs time

The figure shows the agreement between the values calculated from the above presented relation and the values from simulation model

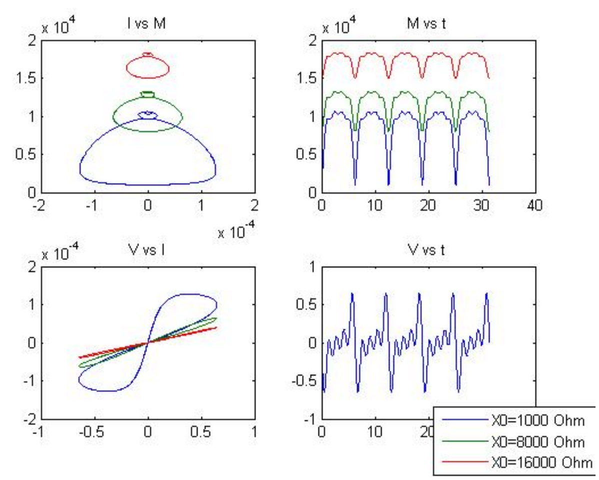

Fig.6 Various plots using Numerical Model

Various plots for voltage signal having fundamental component of frequency ω and three additional harmonics with same amplitude but frequencies as 2ω, 3ω and 4ω respectively were obtained from simulation and are presented here.

CONCLUSION

In presence of additional Harmonics the behavior of the device becomes complicated and it becomes difficult to predict the response due to nonlinear variations in memristance with respect to voltage and frequency. Usage of Fully Symbolic Model was extended from simple sinusoidal source to more complex sources.

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