Blind Source Separation of Underwater Acoustic Signal by Use of Negentropy-based Fast ICA Algorithm

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***Abstract*—Based on in-depth study of independent component analysis (ICA) method and signal independence measure algorithm based on negentropy, the author first conducts pretreatment of centering and whitening the mixed data of underwater acoustic signal, and then applies the negentropy-based fast ICA algorithm to the blind source separation of underwater acoustic signal and performs simulation experiment. The simulation result indicates that the negentropy-based fast ICA algorithm can effectively solve the blind source separation problems in the signal; this also shows that the method has certain universality and has extensive application prospect in the signal processing field.**

***Index Terms*—Centering ,Negentropy, Whitening, Fast ICA.**

I.ICA Question Description

ICA is a newly-developed technology to statistical signal processing with the BSS question, and its processing object is the mixed signal made up of the signal source linear combination of mutually statistics independence. It can find out mutually independent implicit information elements through analyzing the high-order statistical correlation between the multi-dimensional observation data, thus removing high order redundancy between components and finally extracting separated signal components from the mixed signal. This method has been widely used in feature extraction and blind signal processing.

ICA question can be briefly described like this: Suppose there are N sensors with N observation signals $x_i(i=1, 2, \cdots, N)$. Each observation signal is linear blend with M independent source signals: $s_i(i=1, 2, \cdots, M)$, that is $X = AS$. Among them, $X=[x_1, x_2, \cdots, x_N]^T$ and $S=[s_1, s_2, \cdots, s_M]^T$ are mixed signal vector and source signal vector, A is the unknown mixed matrix of $N \times M$. Under such circumstance that the signal source N and the mixed matrix A are both unknown, search the disjunct matrix W and extract mutually independent source signal $\hat{S}=WX$ from the mixed signal, and it is expected that $\hat{S}$ can draw near the real source signal Z better.

Several assumed conditions for solving ICA question are:

(1) Statistical independence between the source signal, and it is the random variable with mean value of zero;

(2) The number of observation signal is not less than the number of the source signal;

(3) Only one source signal can comply with the Gaussian distribution at most;

From the assumed condition (1), we can know that how to measure the statistical independence of the source signal is the key of ICA algorithm. This article makes this non-Gaussian method based on negentropy as the measure of the signal statistical independence.

II. Signal Independence Measure Method Based on Negentropy

Entropy is a basic concept in information theory. Entropy of the random variable can be regarded as the amount of information contained in the observational variable. If the variable is more "random" or immeasurable, the entropy is greater. In the random variable with the same covariance, the entropy of the Gaussian variable is the greatest, namely the so-called entropy maximum theorem. Therefore we often take the Kullback-Leibler (K-L) divergence between the random probability density function $p(x)$ and the Gaussian distribution $p_G(x)$ with the same variance as the measurement of the probability density function non-Gaussian degree, called negentropy and shown by the sign of $J[p(x)]$ or $J(x)$.

$$J[p(x)] = KL[p(x), \ p_G(x)] = \int p(x) \log \frac{p(x)}{p_G(x)} dx \qquad (1)$$

From above formula, we can see that negentropy can be understood as the amount of information of random $p(x)$ and Gaussian distribution $p_G(x)$ with the same secondary moment. Therefore, in theory, negentropy is the most appropriate measurement of non-Gaussian degree.

When negentropy is the non-Gaussian measure, the probability density function of the random variable must be known, but it is unrealistic in practice, so it is not appropriate to use negentropy directly. We often use the approximate value of negentropy to replace it.

The classic approximate negentropy method in ICA is to use higher moment, but the approximate effectiveness of this method is so limited with robustness. The actual algorithm is usually based on the maximum entropy principle, and we can get following approximate negentropy:

$$J(x) \propto \{E[F(x)] - E[F(v)]\}^2 \qquad (2)$$

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Where, $F(\cdot)$ is a non-quadratic term function, and v is a Gaussian variable of zero-mean and unit variance, so select $F(\cdot)$ properly can make above formula have better robustness.

III. DATA PREPROCESSING

In general, the mixed signal data observed has correlation. When conducting the signal blind separation, we often do some preprocessing for the mixed signal. Common preprocessing ways: one way is signal centering, the other is whitening, also called pre-whitening.

A. Signal centering

In most blind separation algorithm, it is required that all components of the source signal is random variable with mean value of zero, which requires removing the mean value of the signal before separation.

The centering process is easy. For the random variable x_i, it shall use $\bar{x}_i = x_i - E(x_i)$ to replace x_i, so as to realize zero mean. In actual calculation, we can use time average to replace statistics average, using the arithmetic mean value to replace mathematical expectation.

Assume $[x_i(1), x_i(2), \cdots, x_i(n)]$ are n samples of random variable x_i, so centering algorithm is

$$\bar{x}_i(t) = x_i(t) - \frac{1}{n}\sum_{t=1}^{n} x_i(t) \qquad t = 1, 2, \cdots, n \tag{3}$$

B. Signal whitening

For some signal blind separation algorithm, whitening is not only a procedure to simplify question, sometimes is also an essential preprocessing process. Whitening process can eliminate correlation among all components of the mixed signal vector, making second order statistics independence among all components of the mixed signal vector after whitening, so as to simplify the extraction process of subsequent independent component; besides, in general, compare whitening data or not, algorithm astringency is better.

So-called signal whitening refers to one N dimension random signal vector X, making output M dimension random signal vector satisfy the condition with correlation matrix as unit matrix, through the linear transformation of one $N \times M$ dimension whitening matrix V. That is

$$Z = VX \tag{4}$$

$$R_z = E[ZZ^T] = I \tag{5}$$

In the blind separation question, conduct pre-whitening for observed mixed signal and put the mixed model $X = AS$ into above formula, get

$$Z = VAS \tag{6}$$

Known correlation matrix is $R_x = E[XX^T]$, because R_x is generally symmetrical and non-negative definite, it can be resolved as follows:

$$R_x = QDQ^T = QD^{1/2}D^{1/2}Q^T \tag{7}$$

In the formula, Q is the characteristic vector matrix of R_x, and it is orthogonal matrix. $D = diag\{\lambda_1, \lambda_2, \cdots, \lambda_N\}$ is the diagonal matrix of corresponding characteristic value, and the correlation matrix of Z is $R_z = VR_xV^T$, then whitening matrix is

$$V = D^{-1/2}Q^T \tag{8}$$

Now it can be ensured

$$R_z = VR_xV^T = D^{-1/2}Q^TQD^{1/2}D^{1/2}Q^T(D^{-1/2}Q^T)^T = I \tag{9}$$

Put $\widetilde{A} = VA$ into $Z = VAS$ again, we can get

$$Z = \widetilde{A}S \tag{10}$$

Since the signal vectors Z and S both meet the condition of correlation matrix as diagonal matrix, $\widetilde{A}$ must be an orthogonal transformation. If regarding Z in above formula as new observation signal, then we can say that whitening makes the original mixed matrix A simplify into a new orthogonal matrix $\widetilde{A}$.

In actual calculation, the correlation matrix of mixed signal can be estimated only through the sample of mixed signal vector, namely using time average to replace statistics average. Assume $[X(1), X(2), \cdots, X(n)]$ are n observation samples of mixed signal vector with zero mean, and now the correlation matrix of mixed signal sample shall be estimated as following formul as

$$R_x = \frac{1}{n-1}\sum_{i=1}^{n} X(i)X^T(i) \tag{11}$$

From above formula, we can know that R_x is a Hermite matrix of nonnegative definition, so its characteristic value is nonnegative real number.

IV. FAST ICA ALGORITHM BASED ON NEGENTROPY

Since Fast ICA algorithm based on negentropy is obtained by fixed point recursive algorithm, also called fixed-point algorithm, it can be applied to any type of data and can be used to analyze high-dimensional data.

Assume the objective function $J(y_i)$ as negentropy of y_i, and from the formula $J(x) \propto \{E[F(x)] - E[F(v)]\}^2$ we can get

$$\nabla u_i \propto \frac{\partial J(y_i)}{\partial u_i} = 2\{E[F(y_i)] - E[F(v)]\}E[\frac{\partial F(y_i)}{\partial u_i}] \tag{12}$$

$$\nabla u_i \propto \{E[F(y_i)] - E[F(v)]\}E[Zf(y_i)] \tag{13}$$

Where, $F(\cdot)$ is a non-quadratic term function, v is a Gaussian variable with zero mean and unit variance, $f(\cdot)$ is the differential coefficient of $F(\cdot)$.

Commonly-used $F(\cdot)$, $f(\cdot)$ and $f'(\cdot)$ are shown in following table:

TABLE I. COMMONLY-USED $F(\cdot)$, $f(\cdot)$ AND $f'(\cdot)$

$F(\cdot)$	$f(\cdot)$	$f'(\cdot)$
$\frac{1}{a_1}\log\cosh a_1 y$	$\tanh a_1 y$	$a_1[1-\tanh^2(a_1 y)]$
$-e^{-y^2/2}$	$ye^{-y^2/2}$	$(1-y^2)e^{-y^2/2}$
y^4	y^3	$3y^2$

Therefore, the fixed point iterative algorithm of adopting negentropy is:

$$u_i(k+1) = E[Zf(y_i)] \quad (14)$$

$$u_i(k+1) \leftarrow \frac{u_i(k+1)}{\|u_i(k+1)\|_2} \quad (15)$$

Practice has proved that the astringency of this algorithm is not so good. Then in order to improve the astringency performance, use Newton iteration algorithm and simplify it by using the nature of Z as whitening data, then we can get the simplified fixed-point iterative algorithm:

$$u_i(k+1) = E[Zf(y_i)] - u_i(k)E\{f'[y_i(k)]\} \quad (16)$$

$$u_i(k+1) \leftarrow \frac{u_i(k+1)}{\|u_i(k+1)\|_2} \quad (17)$$

If you want to extract several information source each time, in principle, and it will be OK as long as you run above-mentioned algorithm some times and extract different $u_i(0)$ each time. But, in order to ensure that the signal sources extracted each time are those that have not been extracted, an orthogonalizing process must precede repeating of above-mentioned algorithm to take out components that have been extracted. As U is orthogonal normalized matrix, Gram-Schmidt orthogonal decomposition method can be used to achieve this purpose. Specific algorithm is as follows:

a) Centering the observation data X, whitening it and getting Z;

b) Assume M as the number of the independent component to be extracted, and make $i=1$;

c) Initialize vector $u_i(0)$ randomly, and meet the principle $\|u_i(0)\|_2 = 1$;

d) Iterate u_i^T of U in line i;

e) Conduct Gram-Schmidt orthogonalization for u_i^T and $u_1^T, \cdots, u_{i-1}^T$ obtained by iteration;

f) Conduct normalization for u_i^T obtained by orthogonalization;

g) If not astringed, return to step (d);

h) Make i add 1, if $i \le M$, return to step (c), or else over.

V. Simulation Verification of Fast ICA Algorithm based on Negentropy

There are three kinds of sound source signal, which are respectively sound from two kinds of marine life recorded by sonar and the sound of paddle rotating collected in lab. Mix them through random mixed weight matrix, then adopt FastICA algorithm based on negentropy to search disjunct matrix and extract source sound signal. Choose $F(y) = -e^{-y^2/2}$ as the approximate non-quadratic term function of negentropy.

The original sound signal and mixed sound signals as shown in Fig. 1:

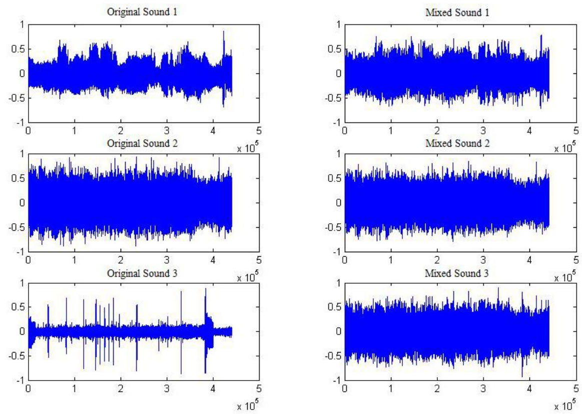

Fig. 1 Original Sound Signal and Mixed Sound Signals

The original sound signal and separated signal as shown in Fig. 2:

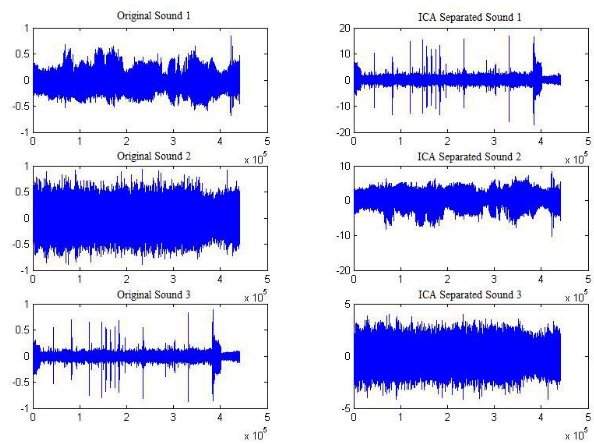

Fig. 2 Original Sound Signal and Separated Sound Signal

VI. Conclusion

The simulation result shows: Fast ICA algorithm based on negentropy has successfully extracted out three different kinds of sound source. However, comparing the extracted signal to the source signal, there exist inverted phenomenon and the serial number of sound source is different as well, which shall be paid attention in application. Secondly, if noise is mixed into mixed signals, especially under the situation with underwater reverberation, the capacity of blind source separation requires further study.

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