DOD-DOA-Polarization Estimation in Large MIMO Radar System Based on Random Matrix Theory

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Abstract—This paper investigates the problem of estimating multiple parameters for large multi-input multi-output (MIMO) radar system in the situation that the numbers of the transmitter and receiver sensors are large that the number of samples could not satisfy the request to confirm that the population covariance matrix would be replaced by the sample covariance matrix. To obtain high-resolution in spatially close angle estimation for multi-target, multiple electromagnetic vector sensors at the receiver are equipped. A novel random matrix theory (RMT)-based approach is proposed to joint estimate the direction-of-departure (DOD), direction-of-arrival (DOA) and two polarization parameters based on two 2D G-MUSIC algorithms. The estimation performance of this paper outperforms that of the conventional algorithms since it has the ability to estimate the parameters more exactly when the number of samples is not sufficient compared with the numbers of the transmitter and receiver sensors. In addition, the exploit of polarization diversity has improved the resolution especially when two or more targets are too close to be distinguished. The simulation results validate the better performance of the proposed method compared with that of the conventional MUSIC algorithm.

Keywords- MIMO radar; random matrix theory; DOD; DOA; polarization; parameter estimation

I. INTRODUCTION

Recently, the research on multi-input multi-output (MIMO) radar and its application in target identification, imaging and localization has been the focus in many literatures [1]. In MIMO radar system, multiple antennas of the transmit array simultaneously send multiple independent signals to illuminate the targets and multiple antennas of the receive array collect the backscattered signals. The bistatic MIMO system can obtain virtual aperture and improve the degree of freedom. Parameter estimation of the targets such as direction-of-departure (DOD) and direction-of-arrival (DOA) is one of the most significant aspects in bistatic MIMO radar, and it plays a critical role in target localization and imaging [2-4]. Since the polarization sensitive array, which consists of electromagnetic vector sensors, has the ability to obtain more information in both space domain and polarization domain than the traditional scalar sensors, it shows better identification performance in targeting. Therefore, joint estimation of the angles and polarizations of the targets in MIMO radar system has been a hot issue [5].

The subspace algorithm, e.g. MUSIC, is widely applied and has high resolution in array signal processing and DOA estimation, which utilizes the eigen-structure of the data covariance matrix of the receive array to obtain the cost function. In conventional subspace approaches in MIMO radar system, we usually assume that the number of array elements remains fixed and the number of snapshots is large which could be infinity. In such situation, the statistic covariance matrix of the observal signals could be substituted by their sample covariance matrix (SCM) to obtain accurate target parameters. However, in large MIMO radar system [6] in which the number of the transmitter and receiver array elements is so large that the number of observations is unable to satisfy the demand given above. Since the observation matrix could be regarded as a large-dimensional random matrix for large MIMO radar system, it is feasible to analyze the problem of parameter estimation using the asymptotic spectrum theory of large-dimensional random matrix as a mathematical tool.

Random matrix theory (RMT) has been applied in kinds of fields, such as wireless communications [7], array signal processing [8-9], etc. In large-dimensional RMT, as the number of the array elements and the number of samples both tend to infinity at the same rate, the empirical spectrum distribution (ESD) of the sample covariance matrix is asymptotic convergence to its limitation spectrum distribution. G-estimator algorithm proposed by Mestre in [10] has shown outstanding performance for DOA estimation.

In this paper, we firstly give the signal model and then propose a novel RMT-based approach to joint estimate the DOD, DOA and two polarization parameters of the targets in large MIMO radar system. The RMT-based G-estimator is extended into multiple parameters estimation in the large MIMO radar application. In the proposed algorithm, we assume that the number of array elements and the number of samples are both finite and comparable in magnitude. It represents high resolution in joint estimating DOD, DOA and polarization of the targets in large MIMO radar system. Since MUSIC algorithm for DOD, DOA and polarization estimation needs a four-dimensional (4D) search which has much computation, we perform two 2D G-MUSIC peak searching to reduce the dimensions based on the eigen-structure of SCM. Simulation results validate that the algorithm given in this paper outperforms the conventional MUSIC algorithm for joint parameter estimation, especially
in the situation that the number of samples is not sufficient compared with the numbers of the transmitter and receiver sensors. Also, the use of polarization diversity can improve the resolution especially when two or more targets are too close to be distinguished.

II. SIGNAL MODEL

The signal model is developed in this section. Assume that the MIMO radar system used here is composed of uniform linear arrays (ULAs) with $M$ conventional scalar sensors at the transmitter and $N$ polarization sensitive sensors at the receiver, all placed spacing half a wavelength. Each sensor at the receiver array consists of spatial independent orthogonal two-electromagnetic dipole. Assume that there are $K$ different far-field narrowband targets distributed in the interest range. The transmit array sends $M$ orthogonal narrowband signals simultaneously and each element at the receive array receives $K$ backscattered signals. Let $\theta_k$, $\phi_k$ and $(\gamma_k, \eta_k)$ respectively stand for the DOD corresponding to the transmitter, the DOA corresponding to the receiver, the polarization angle and polarization phase difference of the $k$-th target. The polarization vector corresponding to the $k$-th target at the receiver array could be formulated as

$$\mu_k = \mu(\phi_k, \gamma_k, \eta_k) = \begin{bmatrix} \cos \phi_k \sin \gamma_k e^{j\eta_k} \\ \cos \phi_k \cos \gamma_k e^{j\eta_k} \end{bmatrix} \in \mathbb{C}^{2 \times 1}$$

where $0 \leq \phi_k, \gamma_k \leq \pi/2$, $-\pi/2 \leq \eta_k \leq \pi$. The received data after matched filtering is

$$y(t) = A b(t) + n(t) \in \mathbb{C}^{N \times M}$$

where $A = [c_1, \cdots, c_K] \in \mathbb{C}^{N \times K}$, with $c_k = c(\theta_k, \phi_k, \gamma_k, \eta_k)$ = $a_1(\theta_k) \otimes a_1(\phi_k) \otimes \mu_k$, $k = 1, 2, \cdots, K$, where $a_1(\theta_k)$ and $a_1(\phi_k)$ denote the transmit array response vector and receive array response vector, respectively, and

$$a_1(\theta_k) = [1, e^{-j \beta_k (L-1) f_d}, e^{-j \beta_k (L-1) f_d}], \in \mathbb{C}^{M \times 1}$$

$$a_1(\phi_k) = [1, e^{-j \phi_k}, e^{-j \phi_k}], \in \mathbb{C}^{N \times 1}$$

where $(\_)^\dagger$ is the transpose, $\otimes$ denotes the Kronecker product.

Denote $b(t) = [b_1(t), \cdots, b_K(t)]^\dagger \in \mathbb{C}^{K \times 1}$ the target signal vector, where $b_k(t) = \beta_k e^{j2 \pi f_d t}$ with $\beta_k$ and $f_d$ representing the scattering coefficient and Doppler frequency of the $k$-th target, respectively. Denote $n(t)$ the noise vector at time $t$, which is assumed to be white Gaussian with zero mean and variance of $\sigma^2$, and uncorrelated with the target signals.

Assume that the parameters of different targets are not identical. Under $L$ snapshots, the observed data could be written as a matrix $Y = [y(1), y(2), \cdots, y(L)] \in \mathbb{C}^{MK \times L}$.

In this paper, let us consider a large MIMO radar system where $2MN$ is comparable with $L$, thus, $Y$ can be regarded as a large-dimensional random matrix.

For subspace method, the covariance matrix $R$ could be formulated as $R = E(Y Y^H)$, where $(\_)^H$ denotes the conjugate transpose. The eigen-decomposition of $R$ could be written as

$$R = \begin{bmatrix} E_x & E_{xy}^\dagger \\ E_{yx} & E_y \end{bmatrix} = \begin{bmatrix} \Lambda_x & 0 \\ 0 & \Lambda_y \end{bmatrix}$$

where $\Lambda_x$ is a $K \times K$ diagonal matrix which contains the $K$ largest eigenvalues of $R$. The signal subspace matrix, $E_x = [e_1, \cdots, e_L] \in \mathbb{C}^{N \times K}$, and the noise subspace matrix, $E_y = [e_{L+1}, \cdots, e_{MN}] \in \mathbb{C}^{N \times (2MN-K)}$, respectively denote the matrix which contains the eigenvectors corresponding to the $K$ largest eigenvalues and the $2MN-K$ smallest eigenvalues, $\sigma^2$, of $R$.

In conventional subspace methods of MIMO radar, the condition of $L >> 2MN$ is given to obtain accurate estimate. While $L$ is not sufficient compared to $2MN$, their performance will become poor.

III. DOD-DOA-POLARIZATION ESTIMATION USING TRADITIONAL 4D MUSIC ESTIMATOR

The basic idea of subspace approaches, which is the core of the traditional MUSIC algorithm, is that any vector lying in the signal space is orthogonal to the noise space, $E_{y}^H c_k = 0$, for $k \in \{1, \cdots, K\}$, i.e., the MUSIC estimator

$$\hat{\xi}_k = \xi(\theta, \phi, \gamma, \eta) = c_k^H E_{y}^H c_k = 0$$

If $L >> 2MN$ ($L$ grows to infinity), the statistical covariance matrix $R$ is well approximated by the SCM,

$$\hat{R} \approx \frac{1}{L} \sum_{l=1}^{L} y(l) y(l)^H$$

then we could utilize the noise subspace of the SCM, $\hat{E}_y = [\hat{e}_{L+1}, \cdots, \hat{e}_{2MN}] \in \mathbb{C}^{N \times (2MN-K)}$, to form a MUSIC estimator. Then the 4D MUSIC spectrum searching can be used to retrieve the arguments $\{\hat{\theta}_k, \hat{\phi}_k, \hat{\gamma}_k, \hat{\eta}_k\}, k = 1, \cdots, K$ which minimize the cost function

$$\hat{\xi}(\theta, \phi, \gamma, \eta) = c_k^H \hat{E}_y^H c_k$$

It is noted that the number of snapshots $L$ often could not meet the condition of $L >> 2MN$ in large MIMO radar system, which affects the efficiency of traditional MUSIC algorithm. We introduce the RMT to dissolve this problem.

IV. DOD-DOA-POLARIZATION ESTIMATION USING TWO 2D G-MUSIC ESTIMATOR

In our method, a large MIMO radar system is considered in which $L$ is not so large compared with $2MN$, i.e., $2MN/L \rightarrow c$ ($c$ is a constant). It is based on the SCM of the observable random matrix $Y \in \mathbb{C}^{2MN \times L}$. At first, we give a RMT theorem of empirical spectrum limitation distribution about spectrum decomposition [10] used in this paper.

Theorem 1: Let $B_N = T_N^H X_N X_N^H T_N \in \mathbb{C}^{N \times N}$, where $X_N \in \mathbb{C}^{N \times L}$. Denote $B_i = \sum_{k=1}^{N} \lambda_k b_k b_k^H$, where $b_k^H b_k = \delta_{kj}$ is the spectral decomposition of $B_i$. Assume $\lambda_1 \leq \cdots \leq \lambda_N$, where
\[ \lambda = (\lambda_1, \cdots, \lambda_N)^T \] are the eigenvalues of \( B_N \). Similarly, denote \( T_k = \sum_{i=1}^K t_i U_i U_i^H \in \mathbb{C}^{N \times N} \), which is a diagonal matrix and has \( K \) distinct eigenvalues, \( 0 < t_1 < \cdots < t_K \), where \( U_i U_i^H = I_n \) with \( U_i \in \mathbb{C}^{N \times n} \) as the eigen-space associated with \( t_k \). Assume that \( u(k; x, y) = x^H U_k y \) for given vectors, \( x, y \in \mathbb{C}^N \), then
\[
u (k; x, y) - \hat{\nu}(k; x, y) \xrightarrow{\alpha \to 0} 0 \tag{10}
\]
as \( N, n \to \infty \) with ratio \( c_n = N / n \to c \), where
\[
\hat{\nu}(k; x, y) = \sum_{i=1}^N \tau_k(i) x^H b_i b_i^H y \tag{11}
\]
and \( \tau_k(i) \) is defined by
\[
\tau_k(i) = \begin{cases} -\phi_k(i), & i \notin \mathbb{N}_k \\ 1 + \psi(k,i), & i \in \mathbb{N}_k \\ \end{cases} \tag{12}
\]
with
\[
\phi(k,i) = \sum_{r \in \mathbb{N}_k} \left( \frac{\lambda_r - \lambda_i}{\lambda_r - \lambda_i} - \frac{\nu_{r,i}}{\nu_{i,i}} \right) \tag{13}
\]
\[
\psi(k,i) = \sum_{r \in \mathbb{N}_k} \left( \frac{\lambda_r - \lambda_i}{\lambda_r - \lambda_i} - \frac{\nu_{r,i}}{\nu_{i,i}} \right) \tag{14}
\]
and \( \nu_{r,i} \leq \cdots \leq \nu_{i,i} \) are the ordered eigenvalues of the matrix
\[
diag(\hat{\lambda}) = \frac{1}{n} \sqrt{\frac{\lambda}{\lambda^T}} \], where \( \hat{\lambda} = (\hat{\lambda}_1, \cdots, \hat{\lambda}_N)^T \).

Based on the idea of Theorem 1 and G-estimator algorithm developed in [10], we bring it into large MIMO radar system to estimate DOD, DOA and polarization of the targets. Based on the signal model given before and the idea of Theorem 1, we have
\[
\hat{\xi}(\theta, \phi, \gamma, \eta) - \xi(\theta, \phi, \gamma, \eta) \xrightarrow{\alpha \to 0} 0 \tag{15}
\]
as both \( 2MN \) and \( L \) grow large with ratio \( c \) being a constant, where a multidimensional G estimator is
\[
\hat{\xi}(\theta, \phi, \gamma, \eta) = c(\theta, \phi, \gamma, \eta)^N \left( \sum_{i=1}^{2MN} \tau(i) \hat{e}_{\xi}^H \right) c(\theta, \phi, \gamma, \eta) \tag{16}
\]
with \( \tau(i) \) defined as
\[
\tau(i) = \begin{cases} 1 + \sum_{k=1}^{2MN-K} \left( \frac{\lambda_k - \hat{\lambda}_i}{\lambda_k - \hat{\lambda}_i} - \frac{\hat{\lambda}_i}{\hat{\lambda}_i} \right), & i \leq 2MN - K \\ - \sum_{k=1}^{2MN-K} \left( \frac{\lambda_k - \hat{\lambda}_i}{\lambda_k - \hat{\lambda}_i} - \frac{\hat{\lambda}_i}{\hat{\lambda}_i} \right), & i > 2MN - K \\ \end{cases} \tag{17}
\]
where \( \hat{\lambda}_i \leq \cdots \leq \hat{\lambda}_{2MN} \) are the ordered eigenvalues of the matrix \( \hat{\lambda} = (\hat{\lambda}_1, \cdots, \hat{\lambda}_{2MN})^T \).

Generally, 4D searching is needed to obtain the minimum of \( \hat{\xi}(\theta, \phi, \gamma, \eta) \) in (16). Here a method to reduce its dimension is given. Since \( c_k = a_k(\theta_k) \otimes a_k(\phi_k) \otimes \mu_k \), it is discovered that the estimator function in (16) can be decomposed into two functions for \( 2D \) peak searching.

In the large dimension case, the noise subspace \( \hat{e}_i \) \( \{i=2MN-K+1, \cdots, 2MN\} \) of \( \hat{R} \) is not identical with the noise subspace \( E_N \) of the conventional MUSIC algorithm. Since
\[
\hat{a}(\theta, \phi) \otimes a(\phi) \otimes a(\phi) \in \mathbb{C}^{MN-1} \), we could obtain the DOD and DOA at first using a 2D G-MUSIC algorithm with the function
\[
\hat{\xi}(\theta, \phi) = \text{det}(\hat{a}(\theta, \phi) \otimes I_2)^H \left( \sum_{i=1}^{2MN} \tau(i) \hat{e}_i^H \right) \left( a(\theta, \phi) \otimes I_2 \right) \tag{19}
\]
where \( \text{det}(\cdot) \) denotes the determinant of a matrix. Upon the estimate of \( (\theta_k, \phi_k) \) is obtained from (19), we calculate two polarization parameter using again the 2D G-MUSIC algorithm with the function
\[
\hat{\xi}(\gamma, \eta) = c(\theta, \phi, \gamma, \eta)^H \left( \sum_{i=1}^{2MN} \tau(i) \hat{e}_i^H \right) c(\theta, \phi, \gamma, \eta) \tag{20}
\]
for \( k \in 1, \cdots, K \).

V. SIMULATION

To evaluate the performance of the proposed method for estimating DOD, DOA and polarization of the targets in large MIMO radar system, computer simulations are conducted.

Example 1: Let’s consider the situation where four targets are located in the interest range. Among them, the first two targets have close DODs and DOAs, similarly with the last two targets. The DOD and DOA of the four targets are \( (\theta_1, \theta_2, \theta_3, \theta_4, \phi_1, \phi_2, \phi_3, \phi_4) = (23, 25, 55, 57) \) and \( (\phi_1, \phi_2, \phi_3, \phi_4) = (40^\circ, 42^\circ, 60^\circ, 62^\circ) \). Polarization of the four targets is \( (\gamma_1, \gamma_2, \gamma_3, \gamma_4, \eta_1, \eta_2, \eta_3, \eta_4) \) = (0.1\pi, 0.25\pi, 0.3\pi, 0.45\pi, 0.3\pi, 0.15\pi, 0.4\pi, 0.25\pi)\). The MIMO radar is composed with arrays with \( M=23 \) sensors at the transmitter and \( N=20 \) sensors at the receiver. The noise is assumed to be white Gaussian. SNR=20dB. The simulation gives the results of proposed DOD-DOA-polarization estimation method using 2D G-MUSIC algorithm, shown in Fig.1 to Fig.4.
From Figs. 1-4, the algorithm given in this paper performs good ability to estimate the DOD, DOA and polarization parameters, even in the situation where two or more targets are located too close to be distinguished.

**Example 2:** To show the advantage of RMT-based algorithm, we calculate the MSE (mean-square error) of DOD and DOA estimation to compare the proposed 2D G-MUSIC algorithm and the traditional 2D MUSIC algorithm. There are two closely spaced targets with $(\theta_1, \theta_2) = (23°, 25°)$, $(\phi_1, \phi_2) = (40°, 42°)$, $(\gamma_1, \gamma_2, \eta_1, \eta_2) = (0.1\pi, 0.25\pi, 0.3\pi, 0.15\pi)$, $M=8$ and $N=8$. SNR is changed from -10dB to 10dB. The MSE can be denoted as

$$MSE = \frac{1}{TK} \sum_{t=1}^{T} \sum_{k=1}^{K} (\hat{\theta}_k^{(t)} - \theta_k)^2$$  \hspace{1cm} (22)

where the Monto-Caro trials are $T=100$, and $\hat{\theta}_k^{(t)}$ denotes the estimate of the parameter $\theta_k$ at the $t$-th trial. Figs. 5 and 6 show the average of MSEs of DOD and DOA estimation v.s. SNR, respectively when $L=150$ and $L=1000$.

The comparison result validates that the proposed algorithm outperforms the traditional 2D MUSIC algorithm when the number of snapshots is not sufficient to meet the demand which confirms the population covariance matrix could be replaced by the SCM. It proves that with the smaller number of snapshots, the 2D G-MUSIC algorithm has the better performance to distinguish targets even when they are too close.
Example 3: We show that comparison of the probability of success between the proposed algorithm and the traditional MUSIC algorithm in multi-parameter estimation. The MIMO radar consists of $M=10$ and $N=12$. There are $L=250$ snapshots. Two targets having the close DOD and DOA parameters $(\theta_1, \theta_2) = (20^\circ, 22^\circ)$, $(\phi_1, \phi_2) = (30^\circ, 32^\circ)$, and polarization $(\gamma_1, \gamma_2, \gamma_3, \gamma_4) = (0.22\pi, 0.23\pi, 0.28\pi, 0.29\pi)$. From Fig.7, with the same SNR, the proposed method can obtain the higher rate of success than the traditional MUSIC algorithm in distinguishing two close targets in space domain. The Monto-Caro trials are $T=100$.

![Figure 7](image_url) The probability of success using G-MUSIC and MUSIC

VI. CONCLUSION

In this paper, we extend the RMT-based G-estimator algorithm into the joint estimation of DOD, DOA and polarization of multiple targets in large MIMO radar system. We develop the signal models used in this paper. Based on the random matrix theory, the DOD-DOA-polarization estimation using two 2D G-MUSIC estimators is given in details. Simulation results validate that the proposed algorithm outperforms the traditional multidimensional MUSIC algorithm in joint parameter estimation. It is commonly applicable in the situation that the number of samples is not sufficient compared with the numbers of the transmitter and receiver sensors. Also, the use of polarization diversity has improved the angle resolution especially when two or more targets are too close to be distinguished.

ACKNOWLEDGMENT

The work is supported by the National Natural Science Foundations of China (No. 61371158 and 61071140).

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