On Complex Picture Hesitant Fuzzy Set and Its Application in Classification Problem

Mahima Poonia, Rakesh Kumar Bajaj, and Varun Shukla

Abstract In this article, we have proposed a new concept of a complex picture hesitant fuzzy set which has advantages over the two theories of picture fuzzy set and hesitant fuzzy set. Later, some algebraic properties related to the proposed theory have also been explained in detail which adds the strong foundation of the concept for further study. An application based on the theory has also been presented which validates the applicability of the concept in solving the problems caused by the fuzziness of the data available.

Keywords Picture fuzzy information · Complex picture fuzzy set · Distance measures · Pattern recognition

1 Introduction

Obtaining solutions to the problems created due to the imprecise and uncertain data encouraged various researchers to work around fuzzy set which has been firstly introduced by Zadeh et al. [1]. During the period of time many extensions have been presented in detail which add various new functions to the theory of membership functions which have already widen the area of obtaining solutions. Some of these theories have been discussed in detail to give the basic idea of the work presented in the current manuscript. Such as Intuitionistic fuzzy set [2] by Atanassov added the non-membership function to fuzzy set and many examples in the field of sciences have proved the advantage of the concept over fuzzy set. Similarly, the neutrality

M. Poonia
ABES Engineering College, Ghaziabad, Uttar Pradesh 201009, India

R. K. Bajaj (✉)
Jaypee University of Information Technology, Waknaghat, Solan, Himachal Pradesh 173234, India
e-mail: rakesh.bajaj@juitsolan.in

V. Shukla
Pranveer Singh Institute of Technology, Kanpur, Uttar Pradesh 209305, India

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membership function has been introduced as an independent function by Smarandache and named it as neutrosophic set [3]. In the neutrosophic set theory, three independent functions (truth, falsity and neutrality) have been introduced which is a generalization of many theories such as classical, fuzzy, intuitionistic and many more. The three independent functions of neutrosophy made it easy to apply the concept to practical life problems related to science [4] and engineering [5]. Later, the favourable interval set was first introduced by Cuong et al., and the set theory is known as hesitant fuzzy set [6]. These sets are generalized versions/extensions of fuzzy sets on two-dimensional Euclidean plane which do not address the phase component present in the form of complex numbers.

This aspect was first noticed by Ramot and the author extended the theory of uncertainty to the complex plane of the unit disc named the theory of complex fuzzy set [7]. A general fuzzy set handles the uncertainty component with the use of only membership function, while the concept under proposition is capable to handle various other types of uncertainty components at the same time. Similarly, several researchers have worked to extend multiple theories (IFS [8], NS [9] and so on) have been extended to the complex plane. The other useful extension named complex picture fuzzy set which was first introduced by Akram et al. extended the concept of picture fuzzy set to complex plane. The problem of robotic agri-farming and green supplier selection has been done in the recent times by making use of the picture fuzzy and q-rung picture fuzzy framework [10–12]. Also, various similarity measures on picture fuzzy sets have been applied in many decision-making problems [13–16]. Various hypersoft extensions of picture fuzzy sets have been done in the literature which are useful to solve many decision-making problems [17–20].

In the present work, we have combined the two important theories form the literature that is picture hesitant fuzzy set and complex picture fuzzy set [21]. Picture hesitant fuzzy set was introduced by Wang et al. [22] and in this the authors have elaborated the concept with the help of some algebraic operations and validated the theory with the help of decision-making application. Now, we have used the theories and extended the concept to complex plane of unit disc. For better understanding of the concept, we have proposed various algebraic operations. Later, the applicability of the proposed theory has been validated with the help of a pattern recognition problem. The pictorial representation of the concept is presented in Fig. 1.

The organization of the current manuscript is in the following manner: Sect. 2 contains the basic definitions of the concept that already exists in literature. The proposed concept is presented in Sect. 3 with few important properties and results. Later, the applicability of the concept has been defined in Sect. 4. Section 5 presents the conclusion of the proposed work.

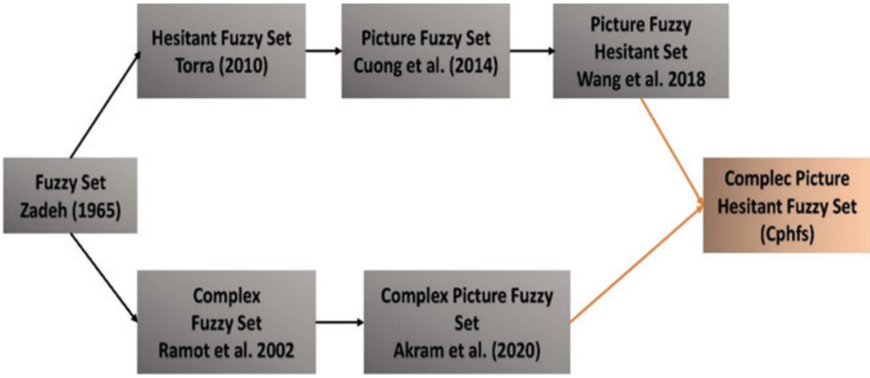

Fig. 1 Extension of complex picture hesitant fuzzy set

2 Preliminaries

In this section of the current manuscript, we have defined a few definitions which will increase the basis of the current theory.

Definition 1 [6] Consider a reference set X, then the hesitant fuzzy set is defined on the reference set in form of h such that it returns a subset of interval [0, 1].

Definition 2 Consider a picture fuzzy set $\varsigma = \{\varrho, \mu(\varrho), \nu(\varrho), \xi(\varrho) | \varrho \in X\}$ on reference set X such that $0 \leq \mu(\varrho) + \nu(\varrho) + \xi(\varrho) \leq 1$. The degree of hesitancy $(\pi(\varrho))$ has been obtained by $\pi(\varrho) = 1 - (\mu(\varrho) + \nu(\varrho) + \xi(\varrho))$. Then, (μ, ν, ξ) is known as picture fuzzy set.

Definition 3 [22] Consider a picture hesitant fuzzy set $\varsigma = \{\varrho, \mu(\varrho), \nu(\varrho), \xi(\varrho) | \varrho \in X\}$ on reference set X such that $0 \leq \sup \mu(\varrho) + \sup \nu(\varrho) + \sup \xi(\varrho) \leq 1$. The degree of refusal can be calculated by $\pi(\varrho) = 1 - (\sup \mu(\varrho) + \sup \nu(\varrho) + \sup \xi(\varrho))$. Then, (μ, ν, ξ) is known as picture hesitant fuzzy set.

Definition 4 [7] Consider a complex fuzzy set $\varsigma = \{\varrho, \mu(\varrho) | \varrho \in X\}$, where the membership function must be complex valued form that is $\mu(\varrho) = \mu_r(\varrho) e^{i2\pi\mu_I(\varrho)} | \mu_r(\varrho), \mu_I(\varrho) \in [0, 1]$. Then, ς is known as complex fuzzy set.

Definition 5 [21] Consider a complex picture fuzzy set $\varsigma = \left\{\varrho, \mu(\varrho) e^{ia(\varrho)}, \nu(\varrho) e^{ib(\varrho)}, \xi(\varrho) e^{ic(\varrho)} | \varrho \in X\right\}$, where $\mu(\varrho), \nu(\varrho), \xi(\varrho) \in [0, 1]$ and $a(\varrho), b(\varrho), c(\varrho) \in [0, 2\pi]$.

Then, it must satisfy the following conditions:

$$\mu(\varrho) + \nu(\varrho) + \xi(\varrho) \leq 1,$$

$$a(\varrho) + b(\varrho) + c(\varrho) \leq 2\pi.$$

Then, the degree of rejection can be calculated by

$$\vartheta(\varrho) = (1 - (\mu(\varrho) + \nu(\varrho) + \xi(\varrho)))\mathrm{e}^{2\pi-(a(\varrho)+b(\varrho)+c(\varrho))}.$$

Definition 6 [23] Consider a complex hesitant fuzzy set $\varsigma = \{\varrho, \mu(\varrho)|\varrho \in X\}$, where the membership function must be complex valued form that is $\mu(\varrho) = \mu_r(\varrho)\mathrm{e}^{i2\pi\mu_I(\varrho)}$ is a subset of unit disc in complex plane.

Then, $0 \leq \max(\mu_r(\varrho), \mu_I(\varrho)) \leq 1$. Thus, $\varsigma = \{\varrho, \mu(\varrho)|\varrho \in X\}$ is known as complex hesitant fuzzy number.

3 Complex Picture Hesitant Fuzzy Set (CPHFS)

In this section of the presented manuscript, we have explored the novel concept of CPHFS and an example of CPHFS has also been presented for better understanding of the concept. Some basic properties related to the concept have also been defined to strengthen the foundation of the concept.

Definition 7 Consider a CPHFS ς on non-empty and finite referential set X.

The CPHFS ς has been denoted by

$$\varsigma = \{\varrho, \mu(\varrho), \nu(\varrho), \xi(\varrho)|\varrho \in X\},$$

where the functions denote the degree of membership ($\mu(\varrho)$), abstinence ($\nu(\varrho)$) and non-membership ($\xi(\varrho)$), respectively. These three functions must be of complex form and denoted by

$$\mu(\varrho) = \mu_r(\varrho) + i\mu_i(\varrho),$$
$$\nu(\varrho) = \nu_r(\varrho) + i\nu_i(\varrho),$$
$$\xi(\varrho) = \xi_r(\varrho) + i\xi_i(\varrho),$$

where all $\mu_r(\varrho), \nu_r(\varrho), \xi_r(\varrho), \mu_i(\varrho), \nu_i(\varrho)$ and $\xi_i(\varrho)$ must lie in interval [0, 1] or is the subset of unit discover complex plane. The degree of refusal is calculated by $\pi(\varrho) = \pi_r(\varrho) + i\pi_i(\varrho)(\pi_r(\varrho) = 1 - \sup(\mu_r(\varrho)) - \sup(\nu_r(\varrho)) - \sup(\xi_r(\varrho)))$. Similarly, for imaginary part of refusal function. Let us consider a CPHF element be $\zeta = \left\{\mu^h, \nu^h, \xi^h \middle| \mu^h \in \mu(\varrho), \nu^h \in \nu(\varrho), \xi^h \middle| \in \xi(\varrho)\right\}$ which we will be using in all the operations related to CPHFS.

Definition 8 Suppose $\mathbb{A} = (\varrho, \mu_{\mathbb{A}}(\varrho), \nu_{\mathbb{A}}(\varrho), \xi_{\mathbb{A}}(\varrho))$ and $\mathbb{B} = (\varrho, \mu_{\mathbb{B}}(\varrho), \nu_{\mathbb{B}}(\varrho), \xi_{\mathbb{B}}(\varrho))$ be two CPHFS. Then,

1. $c(\mathbb{A}) = \{(\varrho, \xi_{\mathbb{A}}(\varrho), \nu_{\mathbb{A}}(\varrho), \mu_{\mathbb{A}}(\varrho))\}$,
2. $\mathbb{A} \cup \mathbb{B} = \{\varrho, \max(\mu_{\mathbb{A}}(\varrho), \mu_{\mathbb{B}}(\varrho)), \min(\nu_{\mathbb{A}}(\varrho), \nu_{\mathbb{B}}(\varrho)), \min(\xi_{\mathbb{A}}(\varrho), \xi_{\mathbb{B}}(\varrho))\}$,
   where
   $\max(\mu_{\mathbb{A}}(\varrho), \mu_{\mathbb{B}}(\varrho)) = \max\big(\mu_r^{\mathbb{A}}(\varrho), \mu_r^{\mathbb{B}}(\varrho)\big) + i\max\big(\mu_i^{\mathbb{A}}(\varrho), \mu_i^{\mathbb{B}}(\varrho)\big)$,
   $\min(\nu_{\mathbb{A}}(\varrho), \nu_{\mathbb{B}}(\varrho)) = \min\big(\nu_r^{\mathbb{A}}(\varrho), \nu_r^{\mathbb{B}}(\varrho)\big) + i\min\big(\nu_i^{\mathbb{A}}(\varrho), \nu_i^{\mathbb{B}}(\varrho)\big)$,
   $\min(\nu_{\mathbb{A}}(\varrho), \nu_{\mathbb{B}}(\varrho)) = \min\big(\nu_r^{\mathbb{A}}(\varrho), \nu_r^{\mathbb{B}}(\varrho)\big) + i\min\big(\nu_i^{\mathbb{A}}(\varrho), \nu_i^{\mathbb{B}}(\varrho)\big)$,
   $\min(\xi_{\mathbb{A}}(\varrho), \xi_{\mathbb{B}}(\varrho)) = \min\big(\xi_r^{\mathbb{A}}(\varrho), \xi_r^{\mathbb{B}}(\varrho)\big) + i\min\big(\xi_i^{\mathbb{A}}(\varrho), \xi_i^{\mathbb{B}}(\varrho)\big)$.
3. $\mathbb{A} \cap \mathbb{B} = \{\varrho, \min(\mu_{\mathbb{A}}(\varrho), \mu_{\mathbb{B}}(\varrho)), \max(\nu_{\mathbb{A}}(\varrho), \nu_{\mathbb{B}}(\varrho)), \max(\xi_{\mathbb{A}}(\varrho), \xi_{\mathbb{B}}(\varrho))\}$,

where

$$\min(\mu_{\mathbb{A}}(\varrho), \mu_{\mathbb{B}}(\varrho)) = \min\big(\mu_r^{\mathbb{A}}(\varrho), \mu_r^{\mathbb{B}}(\varrho)\big) + i\min\big(\mu_i^{\mathbb{A}}(\varrho), \mu_i^{\mathbb{B}}(\varrho)\big),$$

$$\max(\nu_{\mathbb{A}}(\varrho), \nu_{\mathbb{B}}(\varrho)) = \max\big(\nu_r^{\mathbb{A}}(\varrho), \nu_r^{\mathbb{B}}(\varrho)\big) + i\max\big(\nu_i^{\mathbb{A}}(\varrho), \nu_i^{\mathbb{B}}(\varrho)\big),$$

$$\max(\xi_{\mathbb{A}}(\varrho), \xi_{\mathbb{B}}(\varrho)) = \max\big(\xi_r^{\mathbb{A}}(\varrho), \xi_r^{\mathbb{B}}(\varrho)\big) + i\max\big(\xi_i^{\mathbb{A}}(\varrho), \xi_i^{\mathbb{B}}(\varrho)\big).$$

Example 1 The CPHFS $\mathbb{A}$ and $\mathbb{B}$ presented on the reference set X. The two CPHFS have been represented by

$$\mathbb{A} = \{((0.1, 0.3) + i(0.5, 0.6)), ((0.2, 0.7) + i(0.4, 0.8)), ((0.7, 0.1) + i(0.4, 0.6))\},$$

$$\mathbb{B} = \{((0.5, 0.3) + i(0.7, 0.1)), ((0.3, 0.5) + i(0.8, 0.4)), ((0.3, 0.2) + i(0.1, 0.7))\}.$$

Then, applying the operations given in Definition 6:

Proof

1. $\begin{array}{l} c(\mathbb{A}) = \{((0.7, 0.1) + i(0.4, 0.6)), ((0.2, 0.7) + i(0.4, 0.8)), \\ \qquad ((0.1, 0.3) + i(0.5, 0.6))\} \end{array}.$
2. $\begin{array}{l} \mathbb{A} \cup \mathbb{B} = \{((0.5, 0.3) + i(0.7, 0.6)), ((0.2, 0.5) + i(0.4, 0.4)), \\ \qquad ((0.3, 0.1) + i(0.1, 0.6)) \end{array}.$
3. $\begin{array}{l} \mathbb{A} \cap \mathbb{B} = \{((0.1, 0.3) + i(0.5, 0.1)), ((0.3, 0.7) + i(0.8, 0.8)), \\ \qquad ((0.7, 0.2) + i(0.4, 0.7))\} \end{array}.$

4 Similarity Measure on CPHFS

In this section of the current manuscript, a new similarity measure for CPHFS has been systematically proposed.

Definition 9 Let $\mathbb{A}$ and $\mathbb{B}$ be two CPHFS in reference set X. Then, the proposed similarity measure $\mathbb{SM}$ is supposed to satisfy

A. $0 < \mathbb{SM} < 1$;

B. $\mathbb{SM}(\mathbb{A}, \mathbb{B}) = 1$ iff $\mathbb{A} = \mathbb{B}$;

C. $\mathbb{SM}(\mathbb{A}, \mathbb{B}) = \mathbb{SM}(\mathbb{B}, \mathbb{A})$.

Definition 10 Consider $\mathbb{A}$ and $\mathbb{B}$ be two CPHFS in reference set X. We propose the similarity measure as

$$\mathbb{SM}(\mathbb{A}, \mathbb{B}) = \sum_{j=1}^{n} \left(\frac{\begin{array}{c} \mu_r^{\mathbb{A}}(\varrho_j)\mu_r^{\mathbb{B}}(\varrho_j) + \mu_i^{\mathbb{A}}(\varrho_j)\mu_i^{\mathbb{B}}(\varrho_j) + \nu_r^{\mathbb{A}}(\varrho_j)\nu_r^{\mathbb{B}}(\varrho_j) \\ +\nu_i^{\mathbb{A}}(\varrho_j)\nu_i^{\mathbb{B}}(\varrho_j) + \xi_r^{\mathbb{A}}(\varrho_j)\xi_r^{\mathbb{B}}(\varrho_j) + \xi_i^{\mathbb{A}}(\varrho_j)\xi_i^{\mathbb{B}}(\varrho_j) \end{array}}{\max\left\{ \begin{array}{c} \left(\begin{array}{c} \left(\mu_r^{\mathbb{A}}(\varrho_j)\right)^2 + \left(\nu_r^{\mathbb{A}}(\varrho_j)\right)^2 + \left(\xi_r^{\mathbb{A}}(\varrho_j)\right)^2 \\ +\left(\mu_i^{\mathbb{A}}(\varrho_j)\right)^2 + \left(\nu_i^{\mathbb{A}}(\varrho_j)\right)^2 + \left(\xi_i^{\mathbb{A}}(\varrho_j)\right)^2 \end{array} \right), \\ \left(\begin{array}{c} \left(\mu_r^{\mathbb{B}}(\varrho_j)\right)^2 + \left(\nu_r^{\mathbb{B}}(\varrho_j)\right)^2 + \left(\xi_r^{\mathbb{B}}(\varrho_j)\right)^2 \\ +\left(\mu_i^{\mathbb{B}}(\varrho_j)\right)^2 + \left(\nu_i^{\mathbb{B}}(\varrho_j)\right)^2 + \left(\xi_i^{\mathbb{B}}(\varrho_j)\right)^2 \end{array} \right) \end{array} \right\}} \right).$$

Theorem 1 The proposed similarity measure given by Definition 10 must satisfy the conditions given in Definition 9.

Proof

A. This is obvious that $\mathbb{SM}(\mathbb{A}, \mathbb{B}) \geq 0$,

$$\begin{aligned} &\mu_r^{\mathbb{A}}(\varrho_j)\mu_r^{\mathbb{B}}(\varrho_j) + \mu_i^{\mathbb{A}}(\varrho_j)\mu_i^{\mathbb{B}}(\varrho_j) + \nu_r^{\mathbb{A}}(\varrho_j)\nu_r^{\mathbb{B}}(\varrho_j) + \nu_i^{\mathbb{A}}(\varrho_j)\nu_i^{\mathbb{B}}(\varrho_j) \\ &\quad + \xi_r^{\mathbb{A}}(\varrho_j)\xi_r^{\mathbb{B}}(\varrho_j) + \xi_i^{\mathbb{A}}(\varrho_j)\xi_i^{\mathbb{B}}(\varrho_j) \leq \max\Big\{\Big(\left(\mu_r^{\mathbb{A}}(\varrho_j)\right)^2 + \left(\nu_r^{\mathbb{A}}(\varrho_j)\right)^2 \\ &\quad +\left(\xi_r^{\mathbb{A}}(\varrho_j)\right)^2 + \left(\mu_i^{\mathbb{A}}(\varrho_j)\right)^2 + \left(\nu_i^{\mathbb{A}}(\varrho_j)\right)^2 + \left(\xi_i^{\mathbb{A}}(\varrho_j)\right)^2\Big), \\ &\quad \Big(\left(\mu_r^{\mathbb{B}}(\varrho_j)\right)^2 + \left(\nu_r^{\mathbb{B}}(\varrho_j)\right)^2 + \left(\xi_r^{\mathbb{B}}(\varrho_j)\right)^2 + \left(\mu_i^{\mathbb{B}}(\varrho_j)\right)^2 + \left(\nu_i^{\mathbb{B}}(\varrho_j)\right)^2 \\ &\quad\quad +\left(\xi_i^{\mathbb{B}}(\varrho_j)\right)^2\Big)\Big\} \\ &\Rightarrow 0 \leq \mathbb{SM} \leq 1. \end{aligned}$$

B. When $\mathbb{A} = \mathbb{B}$,

Then,

$$\mathbb{S}_{\mathbb{M}}(\mathbb{A},\mathbb{A})$$

$$= \frac{1}{n}\sum_{j=1}^{n}\left(\frac{\begin{array}{c}\mu_r^{\mathbb{A}}(\varrho_j)\mu_r^{\mathbb{A}}(\varrho_j)+\mu_i^{\mathbb{A}}(\varrho_j)\mu_i^{\mathbb{A}}(\varrho_j)+\nu_r^{\mathbb{A}}(\varrho_j)\nu_r^{\mathbb{A}}(\varrho_j)\\ +\nu_i^{\mathbb{A}}(\varrho_j)\nu_i^{\mathbb{A}}(\varrho_j)+\xi_r^{\mathbb{A}}(\varrho_j)\xi_r^{\mathbb{A}}(\varrho_j)+\xi_i^{\mathbb{A}}(\varrho_j)\xi_i^{\mathbb{A}}(\varrho_j)\end{array}}{\max\left\{\begin{array}{c}\left(\begin{array}{c}\left(\mu_r^{\mathbb{A}}(\varrho_j)\right)^2+\left(\nu_r^{\mathbb{A}}(\varrho_j)\right)^2+\left(\xi_r^{\mathbb{A}}(\varrho_j)\right)^2\\ +\left(\mu_i^{\mathbb{A}}(\varrho_j)\right)^2+\left(\nu_i^{\mathbb{A}}(\varrho_j)\right)^2+\left(\xi_i^{\mathbb{A}}(\varrho_j)\right)^2\end{array}\right),\\ \left(\begin{array}{c}\left(\mu_r^{\mathbb{A}}(\varrho_j)\right)^2+\left(\nu_r^{\mathbb{A}}(\varrho_j)\right)^2+\left(\xi_r^{\mathbb{A}}(\varrho_j)\right)^2\\ +\left(\mu_i^{\mathbb{A}}(\varrho_j)\right)^2+\left(\nu_i^{\mathbb{A}}(\varrho_j)\right)^2+\left(\xi_i^{\mathbb{A}}(\varrho_j)\right)^2\end{array}\right)\end{array}\right\}}\right)$$

$$\frac{1}{n}\sum_{j=1}^{n}\left(\frac{\begin{array}{c}\mu_r^{\mathbb{A}}(\varrho_j)\mu_r^{\mathbb{A}}(\varrho_j)+\mu_i^{\mathbb{A}}(\varrho_j)\mu_i^{\mathbb{A}}(\varrho_j)+\nu_r^{\mathbb{A}}(\varrho_j)\nu_r^{\mathbb{A}}(\varrho_j)\\ +\nu_i^{\mathbb{A}}(\varrho_j)\nu_i^{\mathbb{A}}(\varrho_j)+\xi_r^{\mathbb{A}}(\varrho_j)\xi_r^{\mathbb{A}}(\varrho_j)+\xi_i^{\mathbb{A}}(\varrho_j)\xi_i^{\mathbb{A}}(\varrho_j)\end{array}}{\begin{array}{c}\mu_r^{\mathbb{A}}(\varrho_j)\mu_r^{\mathbb{A}}(\varrho_j)+\mu_i^{\mathbb{A}}(\varrho_j)\mu_i^{\mathbb{A}}(\varrho_j)+\nu_r^{\mathbb{A}}(\varrho_j)\nu_r^{\mathbb{A}}(\varrho_j)\\ +\nu_i^{\mathbb{A}}(\varrho_j)\nu_i^{\mathbb{A}}(\varrho_j)+\xi_r^{\mathbb{A}}(\varrho_j)\xi_r^{\mathbb{A}}(\varrho_j)+\xi_i^{\mathbb{A}}(\varrho_j)\xi_i^{\mathbb{A}}(\varrho_j)\end{array}}\right)=1.$$

C. It's obvious that $\mathbb{SM}(\mathbb{A},\mathbb{B})=\mathbb{SM}(\mathbb{B},\mathbb{A})$.

Hence, the proposed similarity measure satisfies all the axioms.

5 Application of SM on CPHFS

Here, we propose the implementation procedure of the proposed measure to the novel concept of CPHFS. Further, the solution to the problem related to pattern recognition has been obtained.

Classification of the problem: Using the proposed similarity measure the unknown material of the building has been obtained. Therefore, initially a team of experts have been selected. Later, on the suggestion of the team of experts various known materials used in the construction of the building have been selected. Further, the unknown data related to the materials have been obtained by using the proposed similarity measure between the known and unknown material data. Then, unknown data will be placed with known material with highest similarity measure.

The steps of the methodology used for obtaining the result of the described problem have been explained with the help of following Fig. 2.

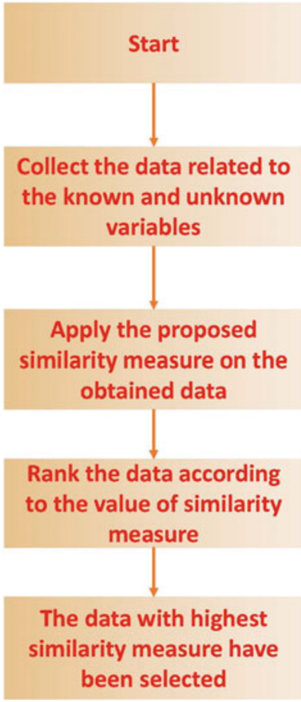

Fig. 2 Methodology used for obtaining the solution

Example 2 Let us consider that the four building materials (steel, mud, stones and bricks) have been denoted by $\alpha_i (1 \le i \le 4)$ and places for the location of building have been denoted by $\theta_i (1 \le i \le 4)$. The data related to known and unknown materials have been listed in Table 1 given.

Step 2. Used the data given in Table 1.

Step 3. Now, from the above data we have obtained the similarity ranking given below:

$\mathbb{SM}(\alpha, \alpha_3) > \mathbb{SM}(\alpha, \alpha_4) > \mathbb{SM}(\alpha, \alpha_2) > \mathbb{SM}(\alpha, \alpha_1)$.

Step 4. Thus, we can say that $\mathbb{SM}(\alpha, \alpha_3)$ has the greatest similarity measure.

This ends our methodology (Table 2).

Table 1 Information data for known and unknown materials

	θ_1	θ_2	θ_3	θ_4
α_1	$[(0.1 + i.2, i.5), (0.3 + i.5, 0.3), (0.3 + i.7, 0.4 + i.2)]$	$[(0.0 + i.4, 0.1 + i.9), (0.3 + i.4, 0.4 + i.8), (0.1 + i.5, 0.2 + i.1)]$	$[(0.2 + i.1, 0.4 + 0.3i), (0.8 + i.6, 0.5 + i.2), (0.4 + i.5, 0.9 + i.4)]$	$[(0.9 + i.0, 0.7 + i.5), (0.2 + i.7, 0.4 + i.6), (0.6 + i.1, 0.2 + i.4)]$
α_2	$[(0.3 + i.0, 0.4 + i.9), (0.3 + i.4, 0.2 + ii.1), (0.1 + i.8, 0.6 + i.1)]$	$[(0.6 + i.2, 0.4 + i.2), (0.1 + i.9, 0.6 + i.7), (0.1 + i.4, 0.1 + i.4)]$	$[(0.5 + i.6, 0.1 + i.6), (0.2 + i.7, 0.2 + i.1), (0.1 + i.4, 0.5)]$	$[(0.9 + i.1, 0.1 + i.6), (0.6 + i.7, 0.4 + i.5), (0.2 + i.5, 0.1 + i.2)]$
α_3	$[(0.1 + i.5, 0.1 + i.2), (0.2 + i.7, 0.5 + i.3), (0.1 + i.5, 0.3 + i.2)]$	$[(0.2 + i.6, 0.2 + i.4), (0.1 + i.7, 0.8 + i.1), (0.6 + i.1, 0.4 + i.2)]$	$[(0.2 + i.0, 0.3 + i.1), (0.4 + i.6, 0.4 + i.4), (0.3 + i.5, 0.1 + i.2)]$	$[(0.7 + i.3, 0.1), (0.2 + i.5, 0.2), (0.4 + i.7, 0.6)]$
α_4	$[(0.8 + i.3, 0.3 + i.2), (0.1 + i.4, 0.1 + i.8), (0.7 + i.5, 0.2 + i.7)]$	$[(0.6 + i.3, 0.2 + ii.3), (0.8 + i.4, 0.2 + i.4), (0.5 + i.1, 0.4 + i.6)]$	$[(0.4 + i.3, 0.4 + i.5), (0.1 + i.5, 0.3 + i.3), (0.1 + i.3, 0.4 + i.1)]$	$[(0.1 + i.5, 0.2 + i.6), (0.2 + i.7, 0.3 + i.2), (0.1 + i.2, 0.1 + i.2)]$
α	$[(0.2 + i.4, 0.2 + i.3), (0.1 + i.3, 0.5 + i.1), (0.1 + i.0, 0.8 + i.4)]$	$[(0.0 + i.5, 0.4), (0.1 + i.4, i.2), (0.8 + i.5, 0)]$	$[(0.2 + i.9, 0.2), (0.4 + i.1, 0.3), (0.7 + i.4, i.2)]$	$[(0.1 + i.3, 0), (0.6 + i.5, 0), (0.4 + i.0, 0)]$

Table 2 Computations based on proposed measures

Similarity measure	$\mathbb{SM}(\alpha, \alpha_i)$
$\mathbb{SM}(\alpha, \alpha_1)$	0.460347
$\mathbb{SM}(\alpha, \alpha_2)$	0.518406
$\mathbb{SM}(\alpha, \alpha_3)$	0.701082
$\mathbb{SM}(\alpha, \alpha_4)$	0.571051

6 Conclusion and Future Work

We have proposed a new concept of complex picture hesitant fuzzy set with some algebraic properties which add the clarity of the concept. This theory has the advantage over the theories of complex picture fuzzy and hesitant fuzzy sets. An example of the proposed measure has also been presented proving the proposed properties. Later, a similarity measure on the CPHFS has also been proposed which proves the usefulness of the concept. In the end, the proposed measure has been validated with the help of an example based on pattern recognition.

Future work: This opens the path for the researchers to obtain various different similarity measure on the proposed concept and the complex nature of the theory extended the theory to the unit disc on complex plane.

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